Spectral Medium Effects on Hadronic Densities

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Outline

- Project goal.
- What particles are we working with?
- Finding particle density.
- Applying a Spectral Function to create a more correct model.
- Applying an in-medium width for the Spectral Function.
- Results / Analysis
- Conclusion
- Acknowledgments

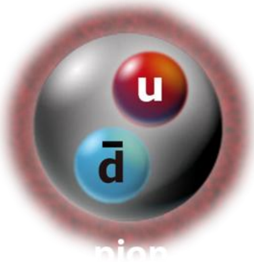
What is the goal of my project?

- The goal of my project is to study the particle density of three particles (i.e. pion, rho meson, and A1 particle) and see how their densities change when a spectral function is applied to try and model density within a medium.
- We hope to find a more accurate model to have a more reliable description for experimental data.

Temperature

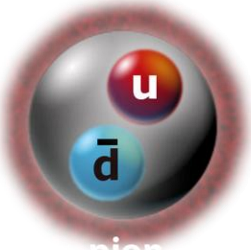
- I am only working with temperatures from about 0 to 200 MeV.
- Once 200 MeV is passed it is no longer considered a hadronic gas but more of a possible quark-gluon plasma.
- To put the temperature I am working with in to perspective 1 eV is about 10^4K which is about 18,000 degrees Fahrenheit. So 1MeV is 10^{10}K.

Pi Meson (Pion)



- Mass of about 140 MeV
- Composed of first generation quarks.
- Has a quark- antiquark pair (e.g. up and anti-down or up and anti-up).
- Lightest of the mesons and is instrumental in understanding the effects of the strong nuclear force at low energies.
- Degeneracy is 3

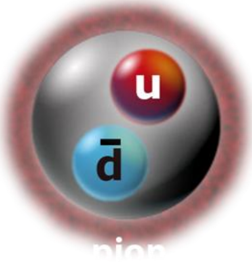
Rho Meson



- Mass of about 770 MeV
- Most prominent resonance when two pions interact.
- Has degeneracy of 9, spin 1.

A1 Particle (Meson)

- Mass of about 1230 MeV
- Resonance of a Rho and a Pion.
- Degeneracy of 9, spin 1.



Particle Density

We start with an equation that gives us number of particles for a certain volume and particle density.

Next we solve for n_i since we are looking to find particle density.

With some manipulation we get our new n_i.

$$N = \int d^3x \, n_i$$

$$n_i = \frac{dN}{d^3x}$$

$$n_i = \int \frac{d^3p}{(2\pi)^3} \frac{g}{e^{E_p/T} \pm 1}$$

$$E_p = \sqrt{p^2 + m^2}$$

- N = number of particles
- n_i = particle density
- d^3x = volume
- T = temperature
- g = degeneracy factor
- p = momentum
- E_p = energy of particle depending on its momentum

Plotting Particle Density

For simplification we denote the Bose distribution as such.

$$f^B(E_p, T) = \frac{1}{e^{E_p/T} - 1}$$

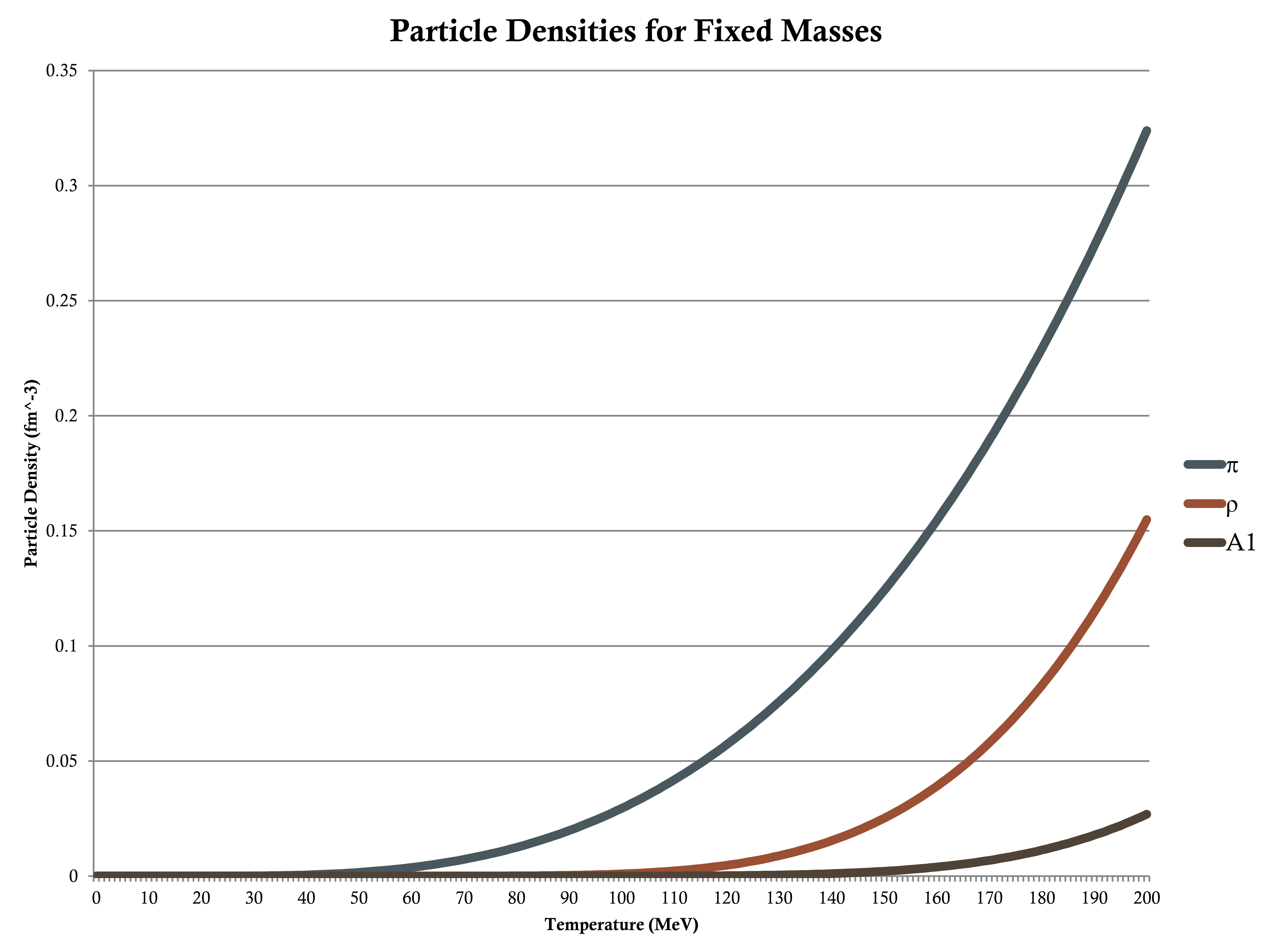
We then use spherical coordinates

$$n_i = g \int_0^\infty \frac{d^3p}{(2\pi)^3} \; f^B(E_p, T)$$

Our last equation is the equation I use to plot particle density.

$$n_i = \frac{g}{2\pi^2} \int_0^\infty p^2 dp \; f^B(E_p, T)$$

Using FORTRAN, once the integral is found I then converted my units from MeV^3 to fm^-3 (fm= fermi or femtometers, 10^-15m). We can now plot P.D. as temperature increases.

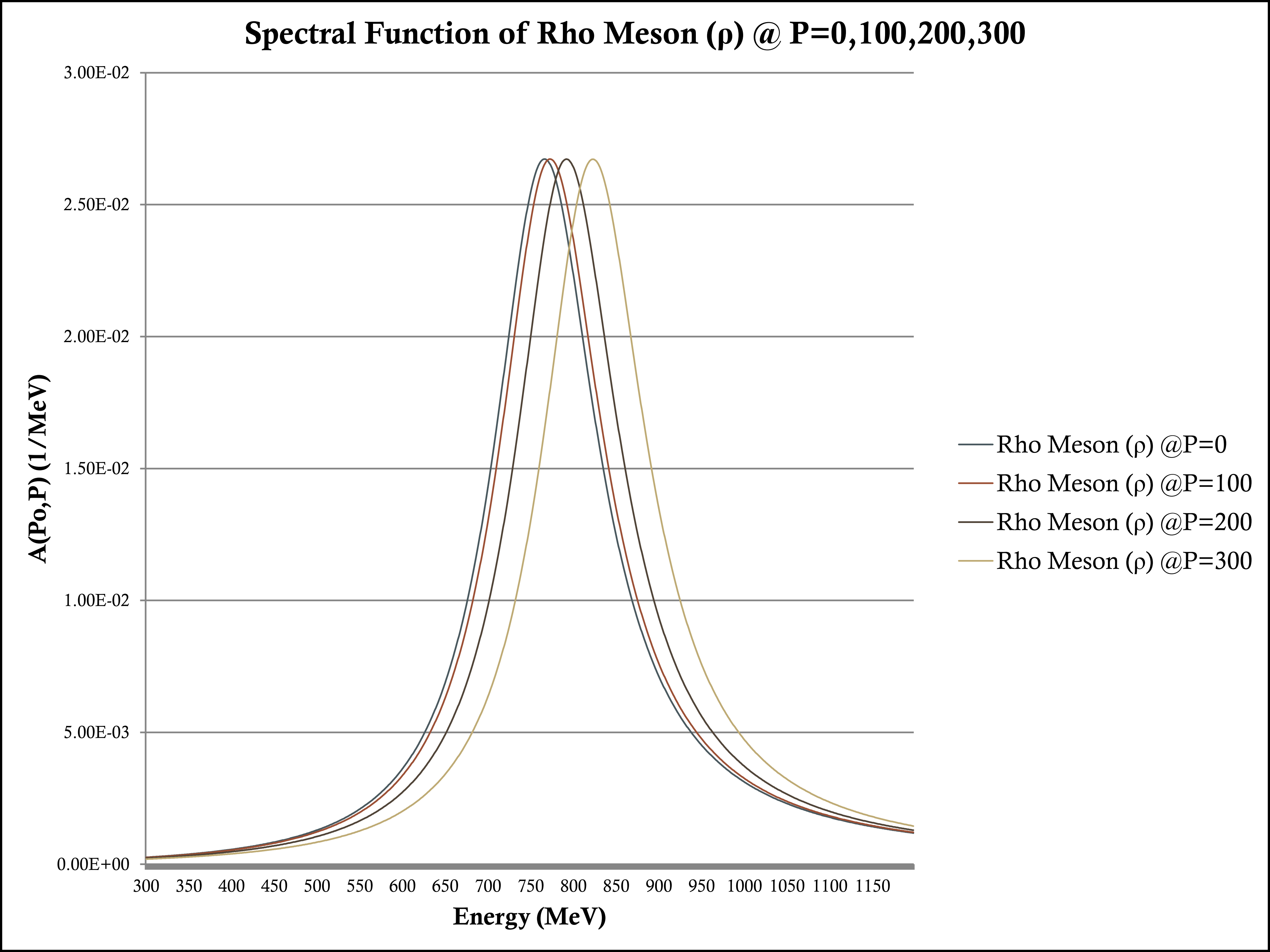

Particle Densities for Fixed Masses
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Particle Density (fm^-3)
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
Temperature (MeV)
π
ρ
A1

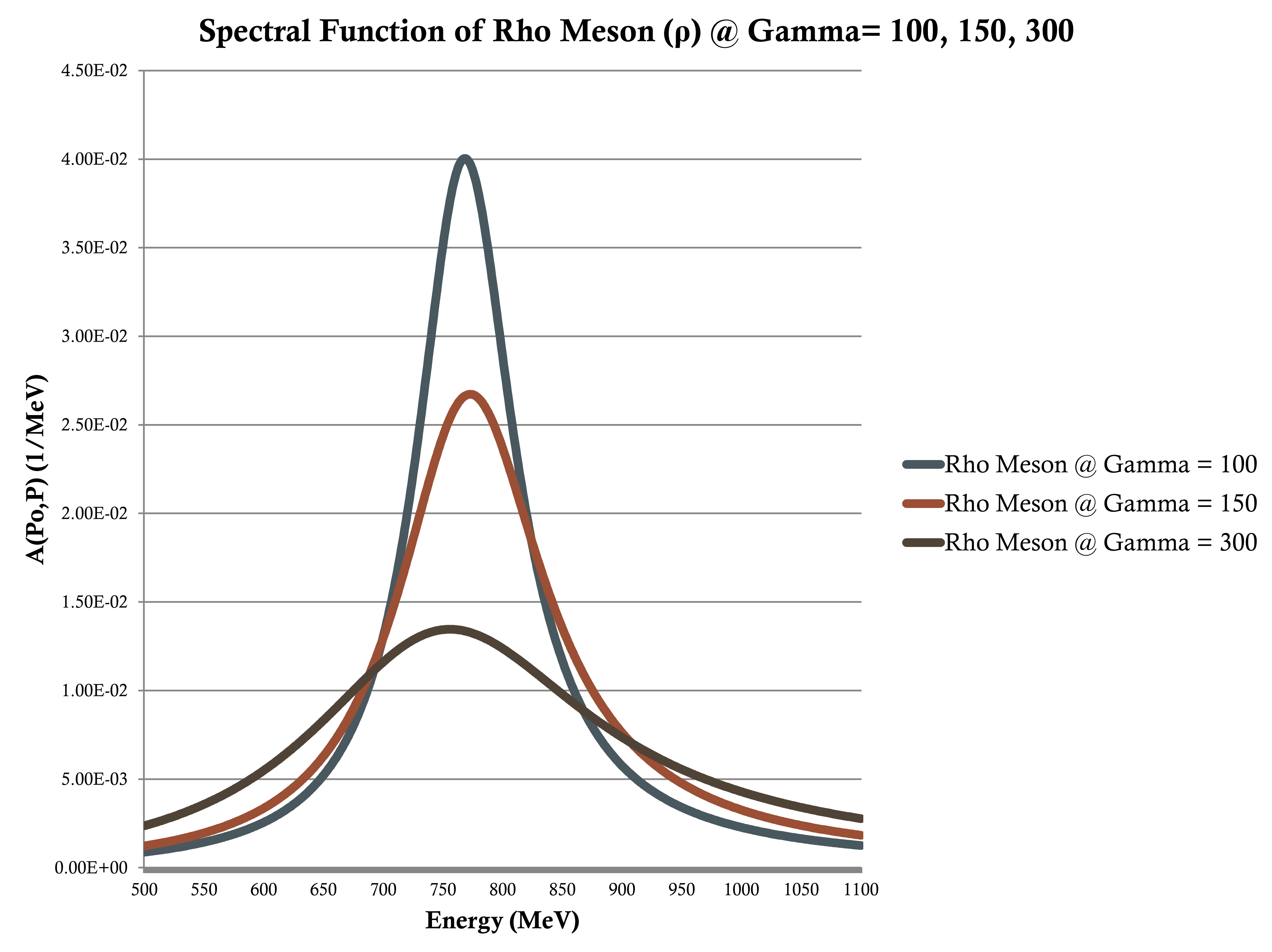
Moving Away From Fixed Masses

- Considered mass to be fixed
- Cant have an accurate model when mass is fixed
- Can make model more correct by adding this spectral function.
- Spectral function dependent upon energy.

$$A_s(p_o, p) = \frac{\Gamma(p_o)}{\left(p_o - E_p\right)^2 + \frac{\left(\Gamma(p_o)\right)^2}{4}}$$

$$\Gamma(p_o) = \Gamma \frac{p_o}{E_p}$$


Spectral Function of Rho Meson (ρ) @ P=0,100,200,300
A(Po,P) (1/MeV)
3.00E-02
2.50E-02
2.00E-02
1.50E-02
1.00E-02
5.00E-03
0.00E+00
300
350
400
450
500
550
600
650
700
750
800
850
900
950
1000
1050
1100
1150
Energy (MeV)
Rho Meson (ρ) @P=0
Rho Meson (ρ) @P=100
Rho Meson (ρ) @P=200
Rho Meson (ρ) @P=300



Spectral Function of Rho Meson (ρ) @ Gamma= 100, 150, 300
4.50E-02
4.00E-02
3.50E-02
3.00E-02
2.50E-02
2.00E-02
1.50E-02
1.00E-02
5.00E-03
0.00E+00
A(Po,P) (1/MeV)
500
550
600
650
700
750
800
850
900
950
1000
1050
1100
Energy (MeV)
Rho Meson @ Gamma = 100
Rho Meson @ Gamma = 150
Rho Meson @ Gamma = 300

Incorporating Spectral Function

We incorporate the new spectral function with a second integral going from momentum (p) to infinity.

We also must account for the changing energy in Bose Distribution function from being a function of E_p to p_o.

$$n_i = \frac{g}{2\pi^2}\int_0^\infty p^2 dp \int_p^\infty \frac{dp_o}{2\pi} A_s(p_o, p)\, f^B(p_o, T)$$

$$f^B(p_o, T) = \frac{1}{e^{p_o/T} - 1}$$

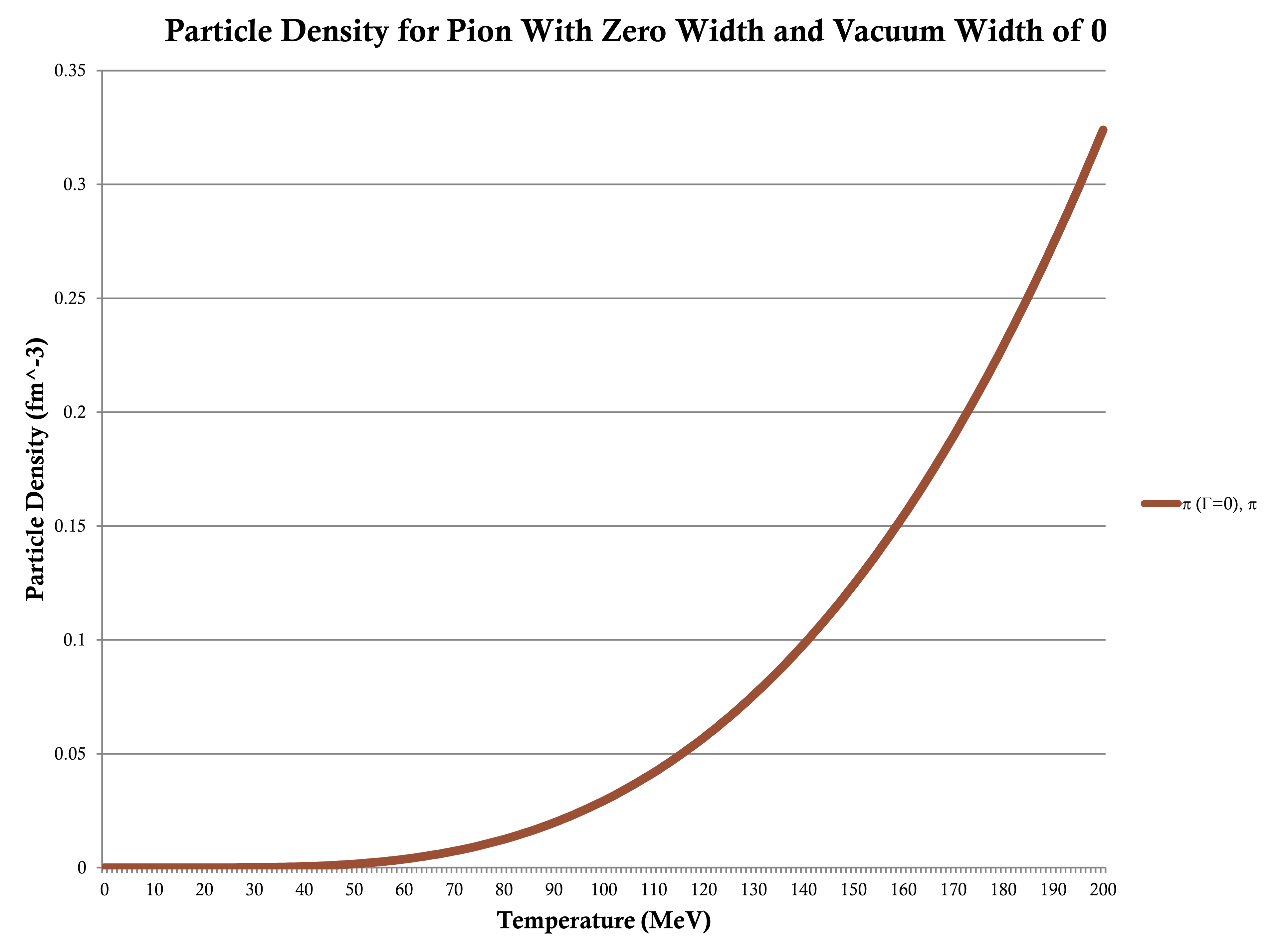
Check Model

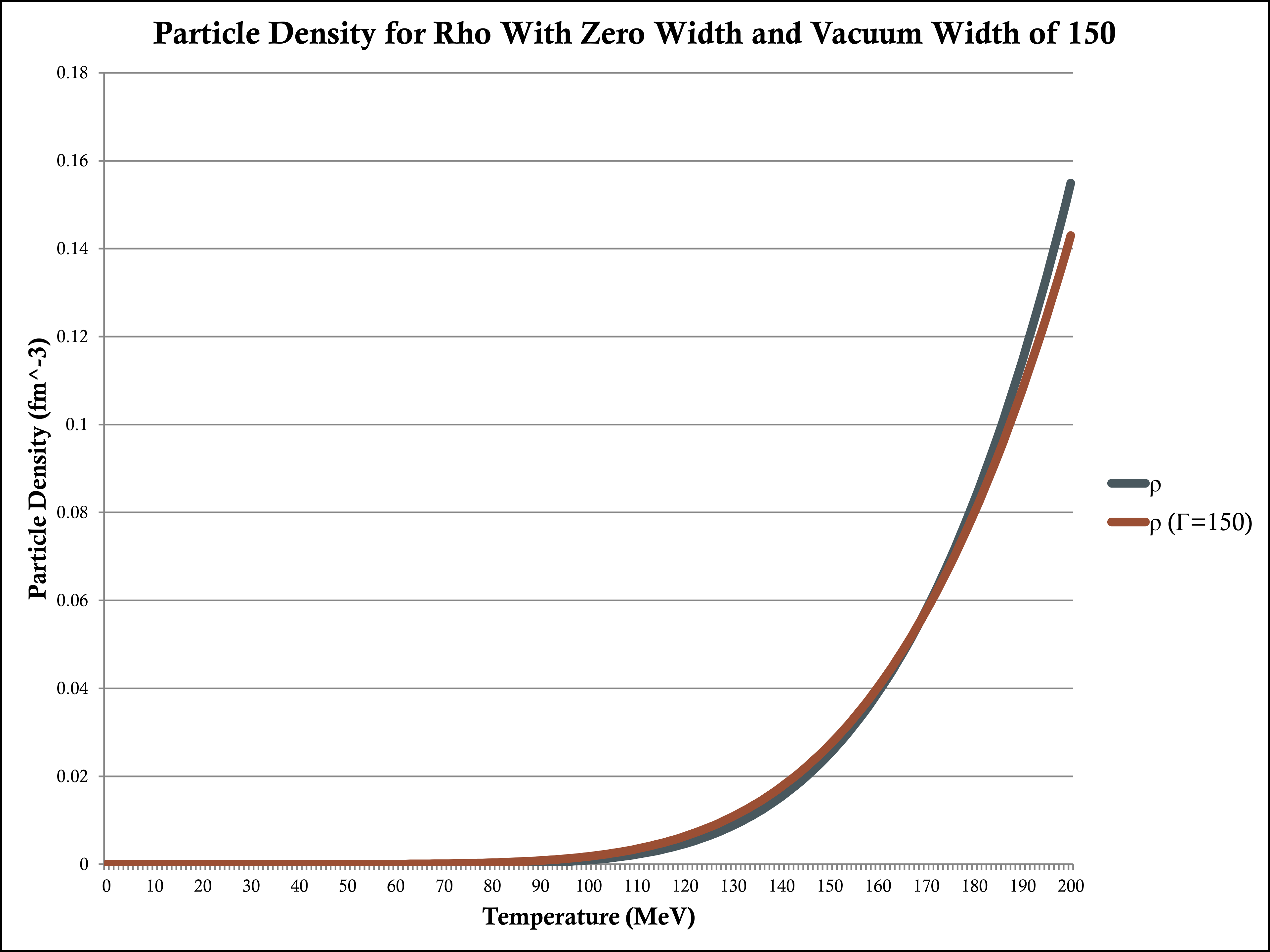
We can check if our new equation is a good model to represent particle density. We just assume that our gamma goes to zero and our spectral function turns into a Dirac delta function.

When integrated you should see that the 2π and $1/2\pi$ cancel out and more importantly all p_o values get replace with an E_p value which gives us our original particle density equation back confirming that our assumptions are correct.

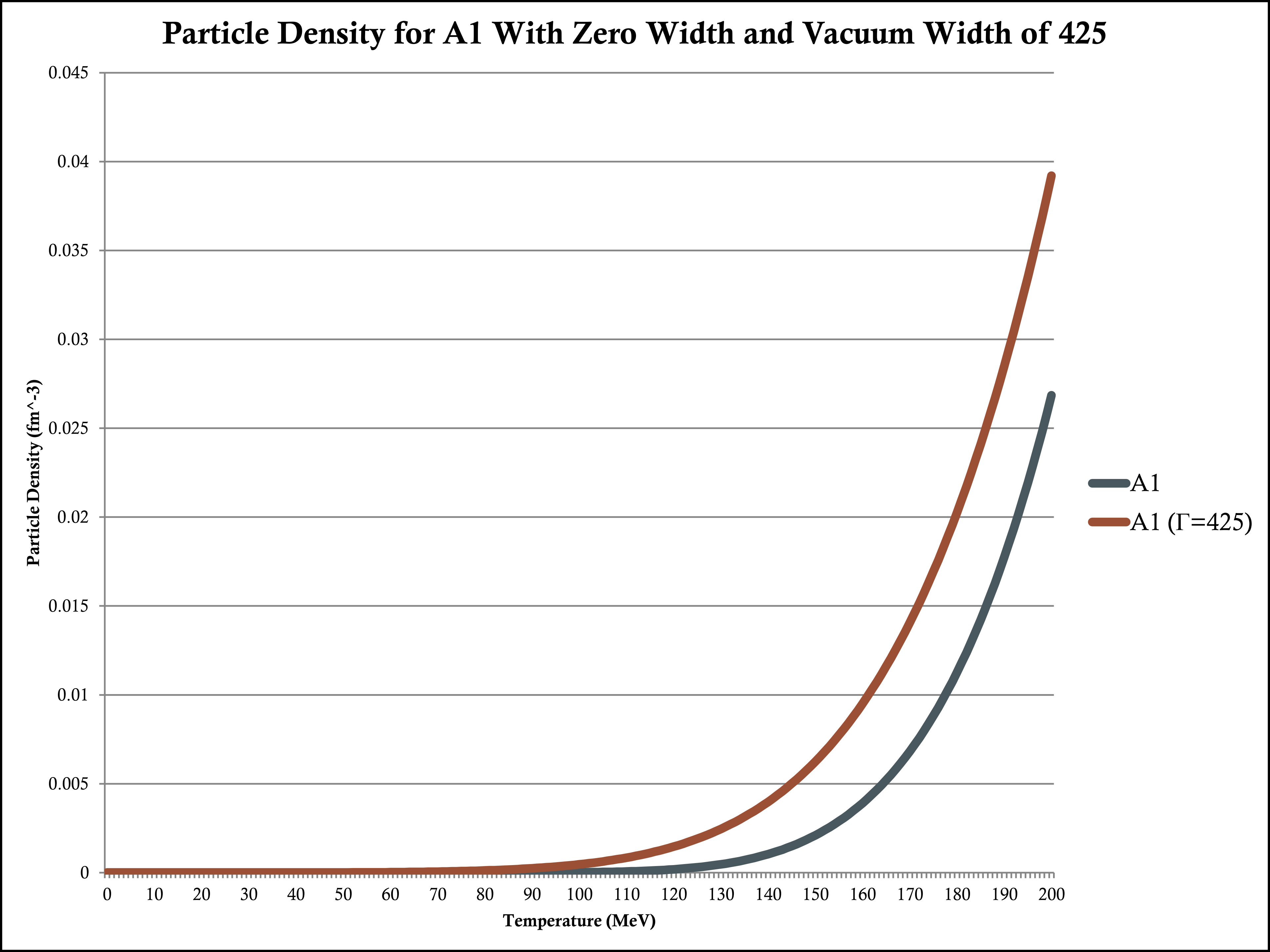
$$n_i = \frac{g}{2\pi^2}\int_0^\infty p^2 dp \int_p^\infty \frac{dp_o}{2\pi}\; 2\pi\; \delta(p_o - E_p)\, f^B(p_o, T)$$

$$n_i = \frac{g}{2\pi^2}\int_0^\infty p^2 dp\;\; f^B(E_p, T)$$


Particle Density for Pion With Zero Width and Vacuum Width of 0
Particle Density (fm^-3)
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 180 190 200
Temperature (MeV)
π (Γ=0), π



Particle Density for Rho With Zero Width and Vacuum Width of 150
Particle Density (fm^-3)
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
Temperature (MeV)
ρ
ρ (Γ=150)



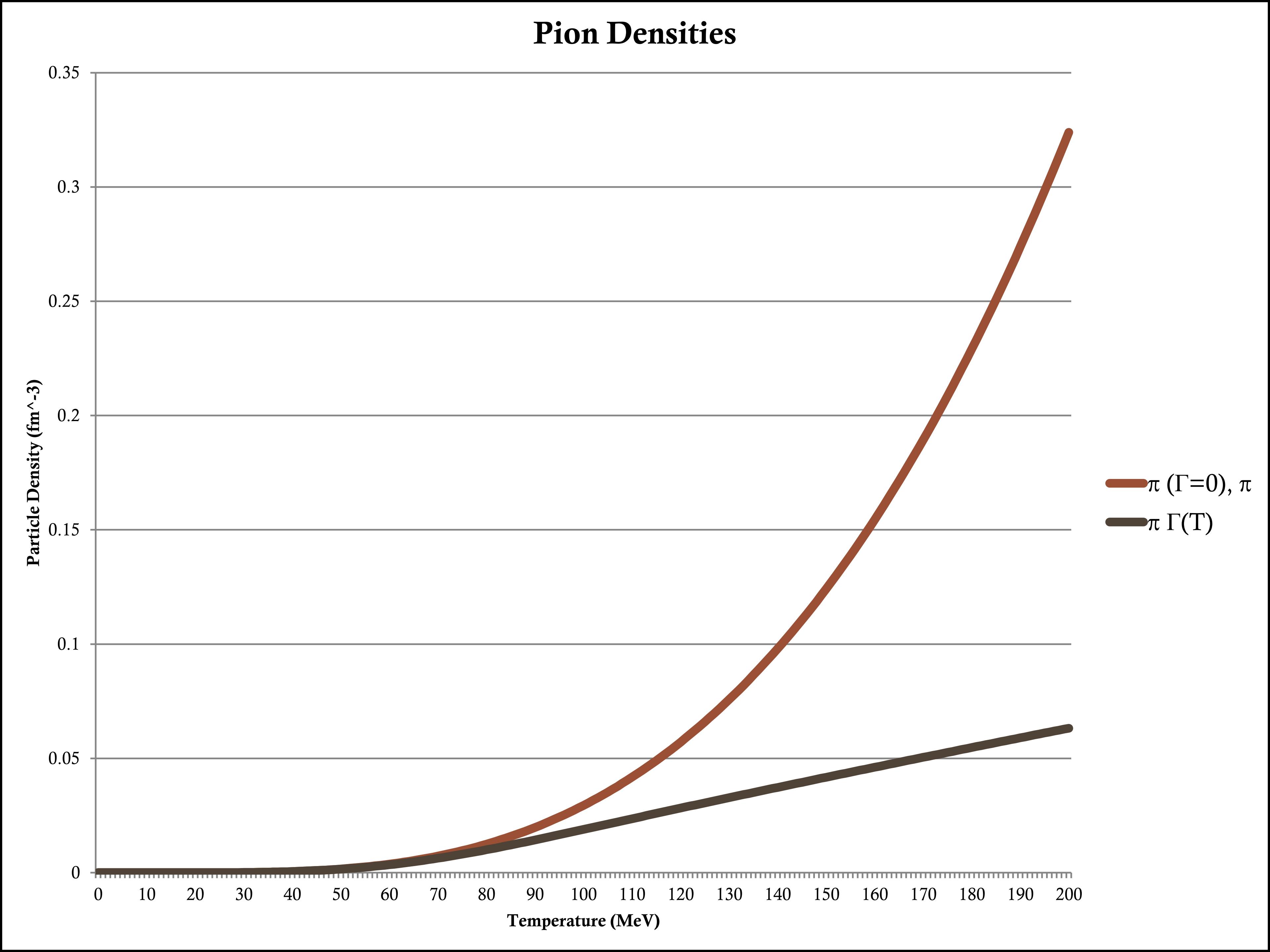
Particle Density for A1 With Zero Width and Vacuum Width of 425
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
Particle Density (fm^-3)
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
Temperature (MeV)
A1
A1 (Γ=425)

How to Improve Gamma

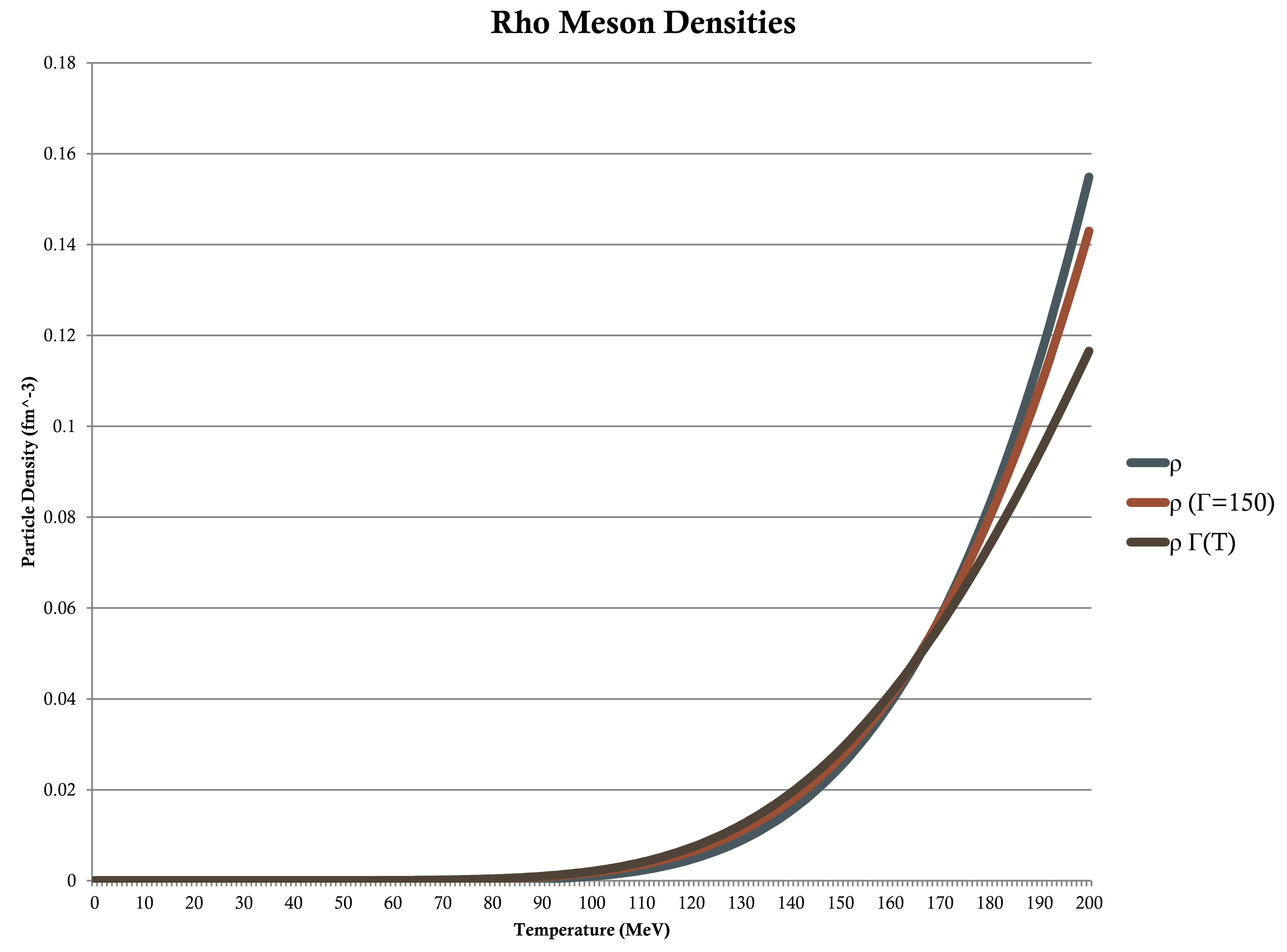
Vacuum width supplemented with temperature dependent in medium contribution.

$$A_s(p_o, p) = \frac{\Gamma(p_o)}{\left(p_o - E_p\right)^2 + \frac{\left(\Gamma(p_o)\right)^2}{4}}$$

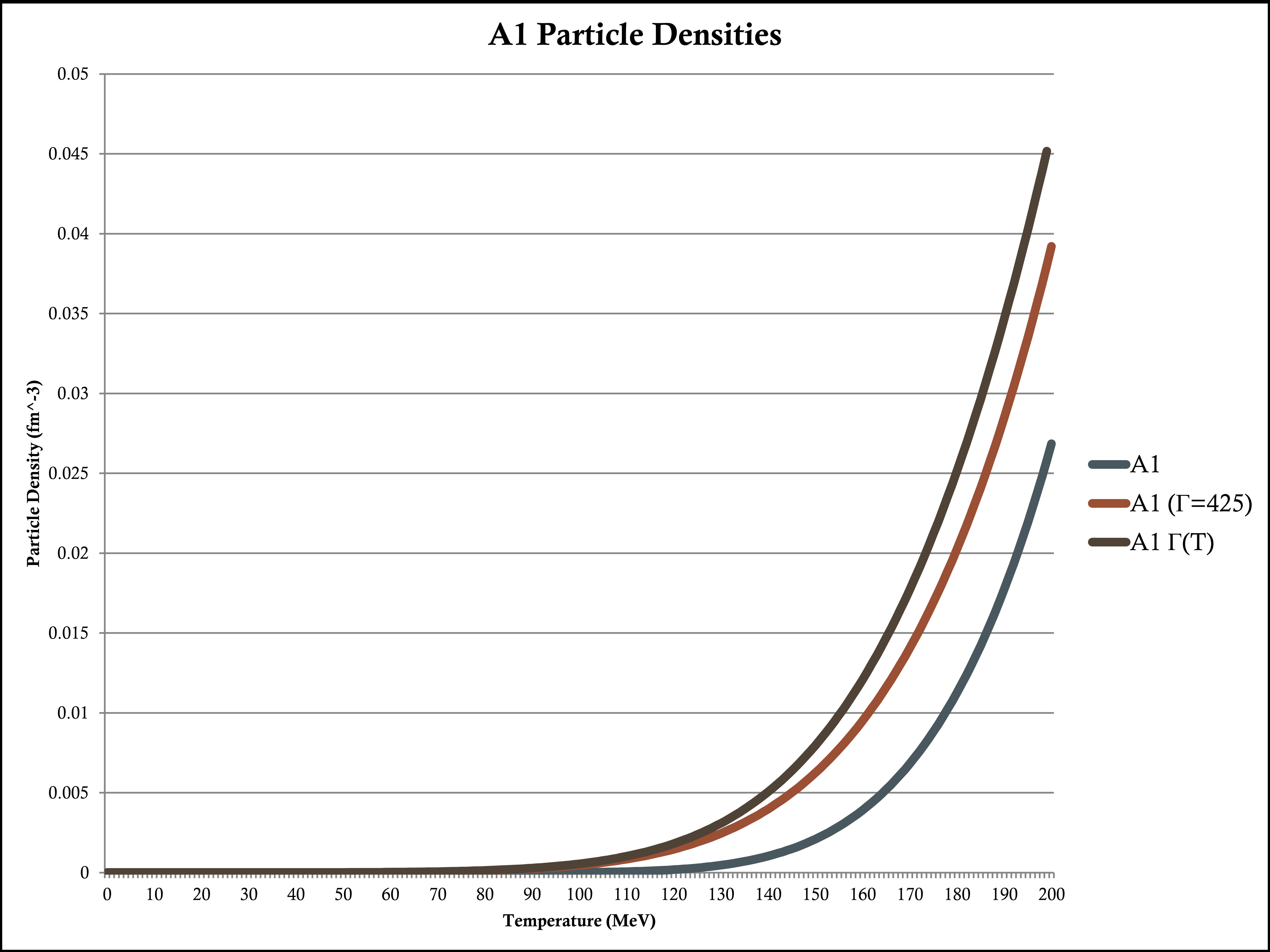
$$\Gamma(T) = \Gamma_{vac} + [\left(\frac{T}{T_o}\right)^3 (\Gamma_{med})]$$

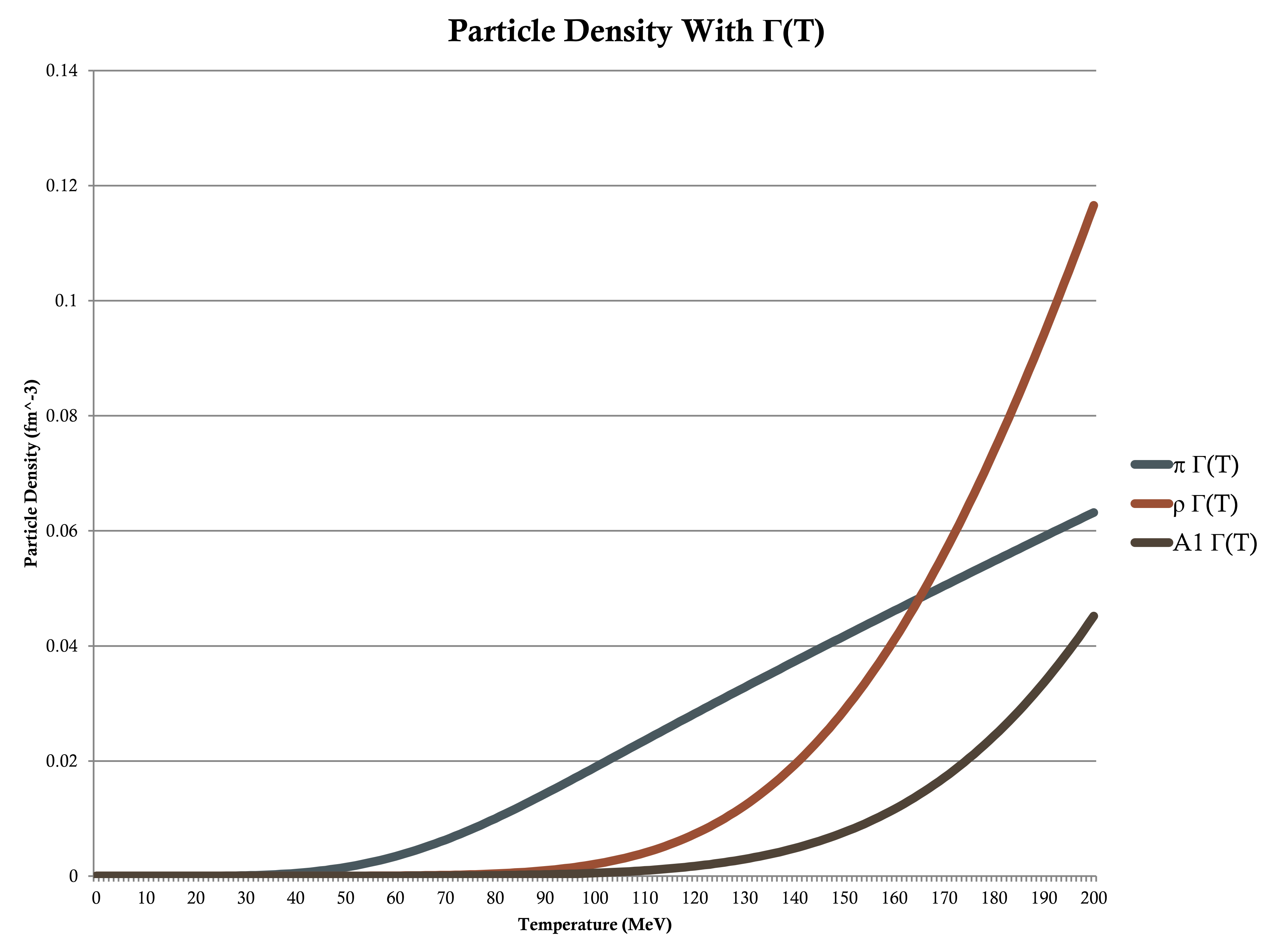


Pion Densities
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Particle Density (fm^-3)
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
Temperature (MeV)
π (Γ=0), π
π Γ(T)

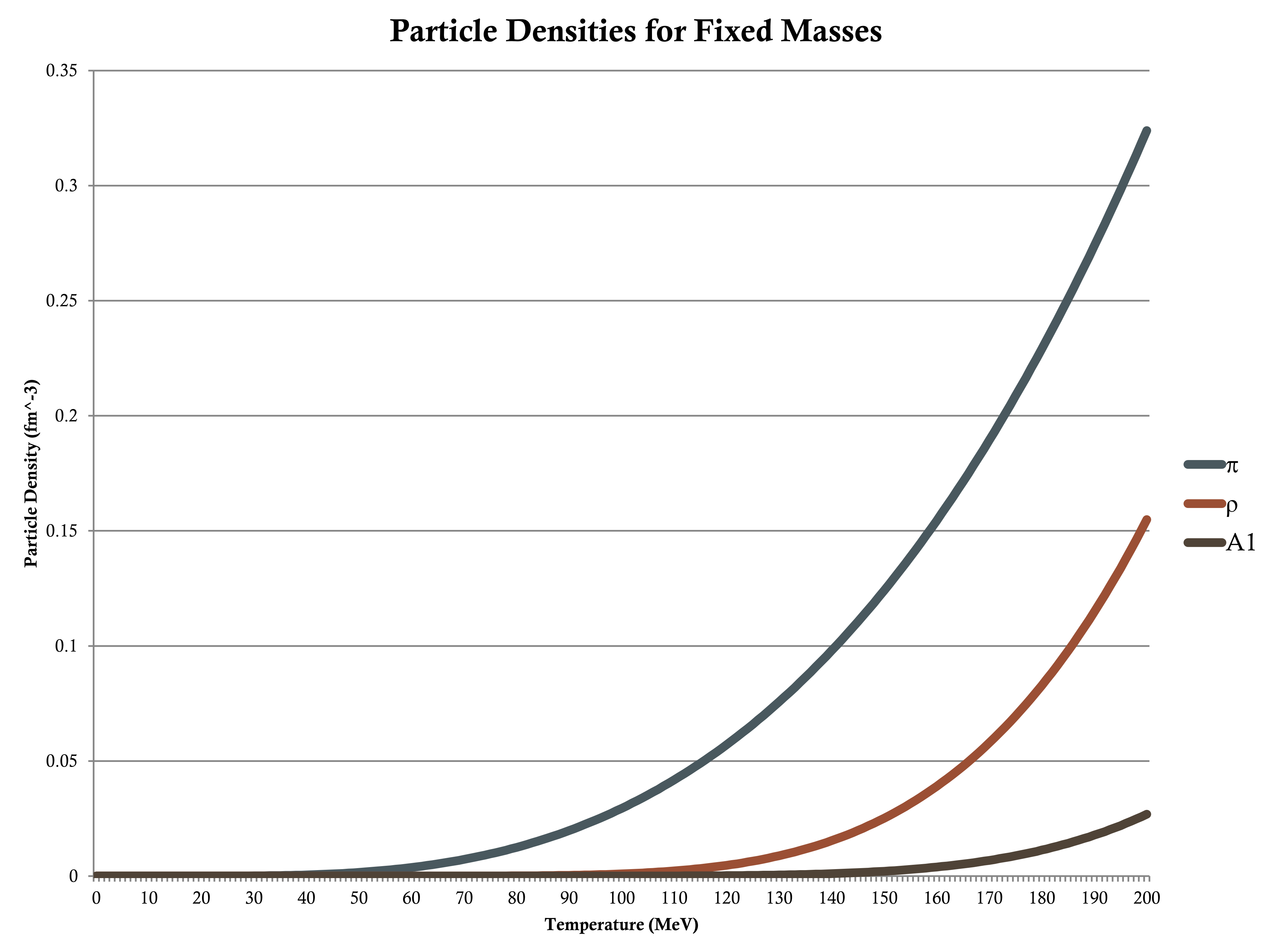


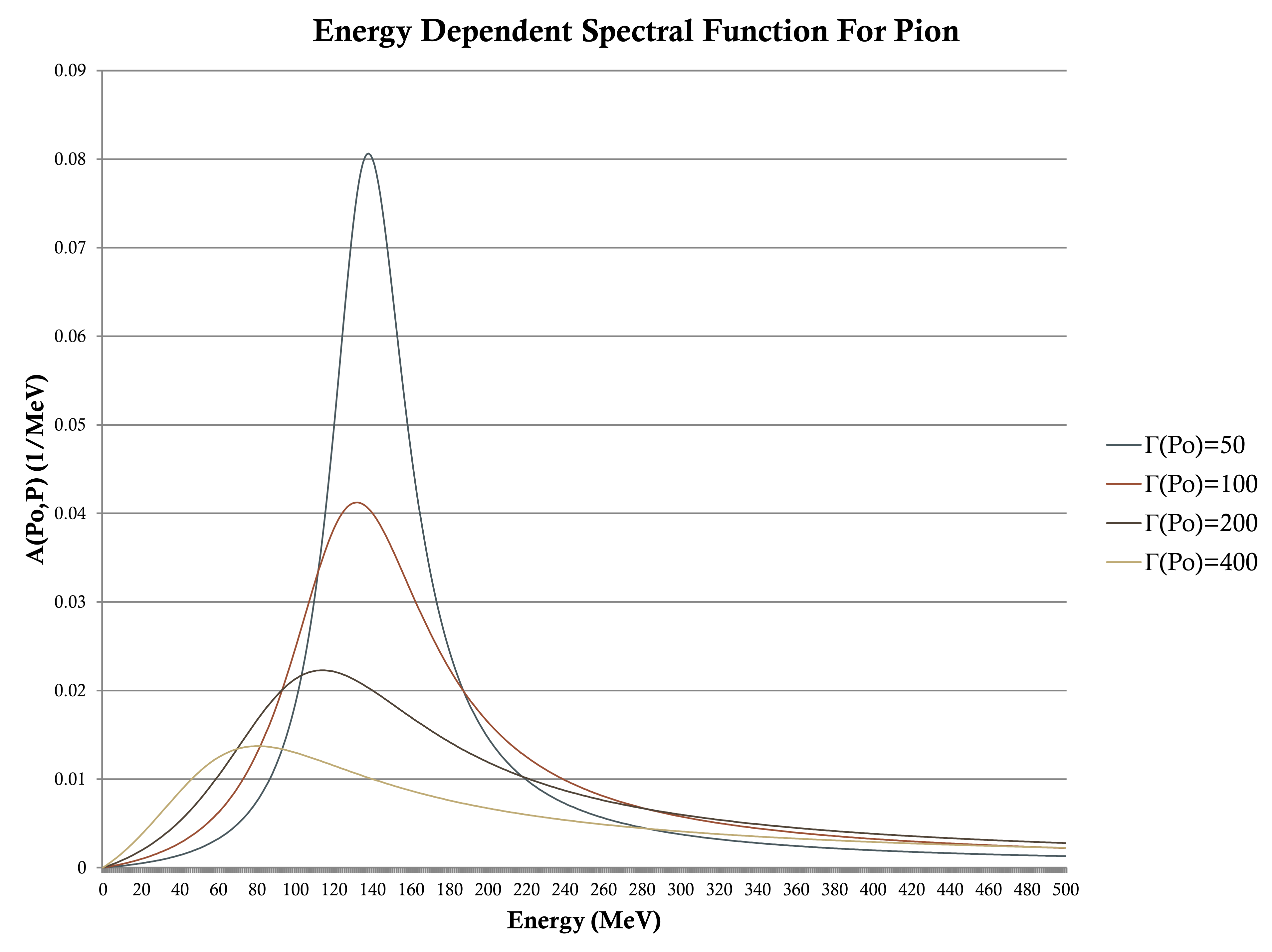
Rho Meson Densities
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
Particle Density (fm^-3)
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
Temperature (MeV)
ρ
ρ (Γ=150)
ρ Γ(T)


A1 Particle Densities
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
Particle Density (fm^-3)
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
Temperature (MeV)
A1
A1 (Γ=425)
A1 Γ(T)

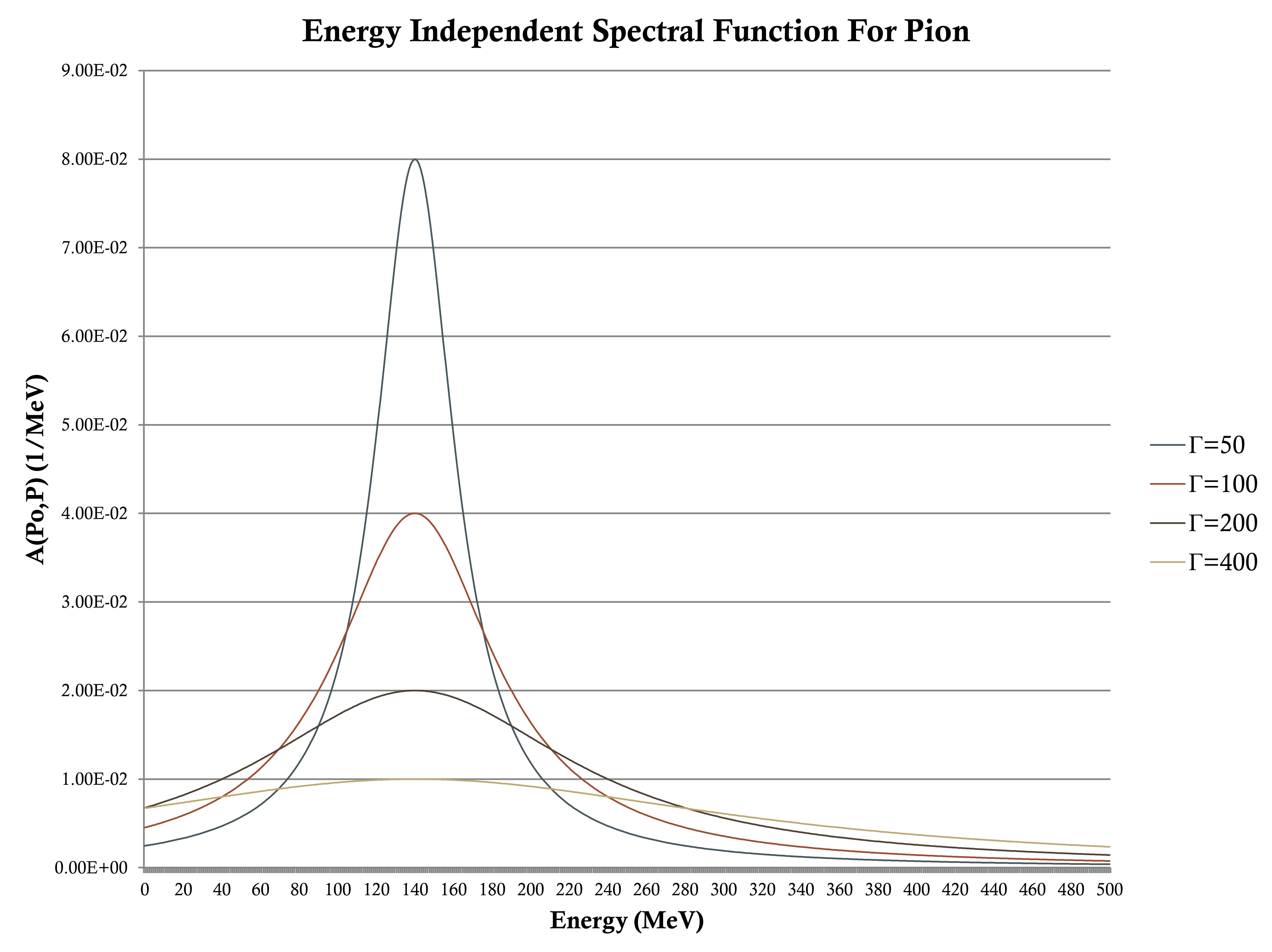


Particle Density With Γ(T)
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
Particle Density (fm^-3)
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
Temperature (MeV)
π Γ(T)
ρ Γ(T)
A1 Γ(T)


Particle Densities for Fixed Masses
Particle Density (fm^-3)
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 180 190 200
Temperature (MeV)
π
ρ
A1



Energy Dependent Spectral Function For Pion
A(Po,P) (1/MeV)
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0 20 40 60 80 100 120 140 160 180 200 220 240 260 280 300 320 340 360 380 400 420 440 460 480 500
Energy (MeV)
Γ(Po)=50
Γ(Po)=100
Γ(Po)=200
Γ(Po)=400


Energy Independent Spectral Function For Pion
A(Po,P) (1/MeV)
9.00E-02
8.00E-02
7.00E-02
6.00E-02
5.00E-02
4.00E-02
3.00E-02
2.00E-02
1.00E-02
0.00E+00
0 20 40 60 80 100 120 140 160 180 200 220 240 260 280 300 320 340 360 380 400 420 440 460 480 500
Energy (MeV)
Γ=50
Γ=100
Γ=200
Γ=400

Conclusion

- When thinking about measuring particle density having a fixed mass is not enough to have an accurate model.
- When we incorporate our spectral function with the vacuum width, it has no effect on the pion density but does change the rho meson and the A1 particle.
- However when we modify our gamma to change with temperature and include an in medium width the pion has a significant change but not so much additional change for the other two particles.
- Some unexpected behavior was found when calculating the new densities with the spectral function .It originates from the energy dependence within the spectral function for the widths (gamma).
- Adding this spectral function into my original equation is an important step to a more accurate description of particle density of hadronic matter within a medium.
- It will thus contribute to a more reliable interpretation of experimental data.

Acknowledgments

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